Basic Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS PAPER 1

PREPARATORY EXAMINATION
MEMORANDUM

SEPTEMBER 2014

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MARKS: 150

TIME : 3 hours

This memorandum consists of 9 pages.

QUESTION 1

1.1.1	$x=0$ or $x=-3$	✓✓ Answers	2
1.1.2	$2x^2+3x-1=0$ $=\frac{-(3)\pm\sqrt{(3)^2-4(2)(-1)}}{2(2)}$ $=0{,}28$ *or* $-1{,}78$	✓ equation in std. form ✓ substitution ✓✓ answers	4
1.1.3	$3x^2-5x-2<0$ $(3x+1)(x-2)<0$ $-\frac{1}{3}<x<2$	✓ quadratic form ✓factors ✓✓critical values and inequality	4
1.1.4	$\frac{x}{2}+3=16-8x+x^2$ $x+6=32-16x+2x^2$ $2x^2-17x+26=0$ $(2x-13)(x-2)=0$ $x=\frac{13}{2}$ or $x=2$ n/a	✓ squaring both sides ✓ trinomial ✓ factors ✓ both answers ✓ selection	5
1.2	$\frac{\sqrt{5^{2013}}}{\sqrt{5^{2011}\left(\sqrt{5^4}-1\right)}}$ $=\frac{\sqrt{5^2}}{24}$ $=\frac{5}{24}$	✓ factorising denominator ✓ simplifying ✓answer	3
1.3	$y=-2x-2$ $8x(-2x-2)+42=2x^2+(-2x-2)$ $18x^2+14x-44=0$ $9x^2+7x-22=0$ $(9x-11)(x+2)=0$ $x=\frac{11}{9}$ *or* $x=-2$ $y=\frac{-40}{9}$ *or* $y=2$ $\left(1\frac{2}{9};-4\frac{4}{9}\right)$ or $(-2;2)$	✓ for y as subject ✓ substitution of y ✓ std. form ✓factors ✓x values ✓y values	6
			[24]

QUESTION 2

2.1.1	11 ; 16	✓✓ answers	2
2.1.2	$a=2;\ d=3;\ n=20$ $S_{20}=\frac{20}{2}[2+59]$ or $\frac{20}{2}[2(2)+3(20-1)]$ $=610$ $a=2;\ r=2;\ n=20$ $S_{20}=\frac{2(2^{20}-1)}{2-1}$ $=2097150$ $\therefore S_{40}=2097760$	✓ for a, d and n values ✓ for substitution into formula ✓ 610 ✓ for a, r and n values ✓ for substitution into formula ✓2097150 ✓answer	7
2.2	$S_n=a+ar+ar^2+...+ar^{n-1} \quad \rightarrow \quad (1)$ $rS_n=ar+ar^2+...+ar^n \quad \rightarrow \quad (2)$ $(2)-(1):$ $rS_n-S_n=ar^n-a$ $S_n(r-1)=a(r^n-1)$ $\therefore S_n=\frac{a(r^n-1)}{r-1}$	✓ for equation (1) ✓ for equation (2) ✓ subtraction on LHS and RHS ✓ factorising	4
2.3.1	$T_n=9x^2\left(\frac{2}{3}x\right)^{n-1}$	✓✓ answer	2
2.3.2	$-1<r<1$ $-1<\frac{2}{3}x<1$ $-\frac{3}{2}<x<\frac{3}{2}$	✓ for condition ✓ for substitution of r value ✓ answer	3
2.4	$\frac{a}{1-r}=16 \quad \rightarrow (1)$ $a+ar=12 \quad \rightarrow (2)$ $From\ (1): \quad a=16(1-r) \rightarrow (3)$ $Subst.\ (3)\ \ into\ \ (2)$ $16(1-r)(1+r)=12$ $16-16r^2=12$ $r^2=\frac{1}{4}$ $r=\frac{1}{2}$ $a=8$	✓ equation (1) ✓ equation (2) ✓ for a in terms of r ✓substitution ✓simplifying ✓ $r^2=\frac{1}{36}$ ✓r value ✓a value	8
			[26]

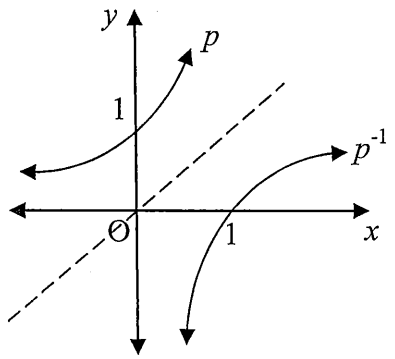
QUESTION 3

3.1	$p=1$ $q=1$ $y=\frac{a}{x+1}+1$ *Now subst.* $P(-2;-2)$: $-2=\frac{a}{-2+1}+1$ $\therefore a=3$	✓ p value ✓ q value ✓ subst. p, q and point P ✓ a value	4
3.2	$\frac{3}{x+1}+1=0$ $x+1=-3$ $x=-4$ $T(-4;0)$	✓for $y=0$ ✓for $x+1=-3$ ✓answer	3
3.3	$1=-(-1)+c$ $\therefore c=0$ OR $y=-(x+p)+q$ $y=-(x+1)+1$ $y=-x$ $\therefore c=0$	✓✓subst. of point A ✓ answer ✓substitution of p and q values ✓equation of line of symmetry ✓answer	3 3
			[10]

QUESTION 4

4.1	$x^2-2x-3=0$ $(x+1)(x-3)=0$ $x=-1 \quad or \quad x=3$ $B(3;0)$ $C(0;-3)$	✓$y=0$ ✓factors ✓coordinates of B ✓coordinates of C	4
4.2	$Axis\ of\ symmetry: \quad x=1$ $Minimum\,Value: \quad y=-4$ $D(1;-4)$	✓axis of symmetry value ✓minimum value ✓ answer	3
4.3	$a=1 \quad and \quad q=-3$	✓✓ answers	2
4.4	$ED = (x-3)-(x^2-2x-3)$ $=-x^2+3x$ $ED=-2(1)+3=2$ OR $ED = y_E - y_D$ $=(1-3)-(-4)=2units$	✓length of ED in terms of x ✓ answer ✓ $h(1)-g(1)$ ✓answer	2 2
4.5	$k=-4$	✓ answer	1
4.6	$-1<x<1 \ or \ x>3$	✓✓✓answers	3
			[15]

QUESTION 5

5.1	$y=\log_5 x$	✓✓ answer	2
5.2		✓✓ graph of p ✓✓graph of p^{-1}	4
5.3	$\log_5 x=1$ $x=5$ $0<x\leq 5$	✓✓for end points and inequality	2
5.4	$t(x)=5^{x-2}+3$	✓for 5^{x-2} in equation ✓for +3 in equation	2
			[10]

QUESTION 6

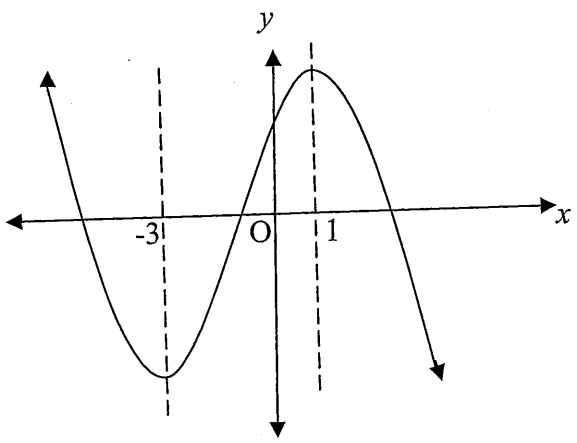
6.1	$A = P(1+i)^n$ $69079{,}71 = 15000(1+0{,}165)^n$ $n = \dfrac{\log\dfrac{69079{,}51}{15000}}{\log(1+0{,}165)}$ $= 10$ years	✓formula ✓substitution ✓making n subject ✓answer	4
6.2.1	R132 000	✓answer	1
6.2.2	$P = \dfrac{x[1-(1+i)^{-n}]}{i}$ $132000 = \dfrac{x\left[1-\left(1+\dfrac{0{,}115}{12}\right)^{-96}\right]}{\dfrac{0{,}115}{12}}$ $\dfrac{132000 \times \dfrac{0{,}115}{12}}{\left[1-\left(1+\dfrac{0{,}115}{12}\right)^{-96}\right]} = x$ $R2109{,}28$	✓formula ✓✓substitution ✓ x as subject ✓ answer	5
6.3	Balance on Loan after the 154th payment $B = A - F_v$ $= 300000\left(1+\dfrac{0{,}18}{12}\right)^{154} - \dfrac{5000\left[\left(1+\dfrac{0{,}18}{12}\right)^{154}-1\right]}{\dfrac{0{,}18}{12}}$ $= R2970925{,}498 - R2967694{,}998$ $= R3230{,}50$ Final payment will be $3230{,}50\left(1+\dfrac{0{,}18}{12}\right)$ $= R3278{,}96$	✓substitution of A ✓substitution of F_v ✓R3230,50 ✓for one month's interest ✓ answer	5
			[15]

QUESTION 7

7.1	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{\frac{4}{x+h}-\frac{4}{x}}{h}$ $=\lim\limits_{h\to 0}\dfrac{\frac{4x-4(x+h)}{x(x+h)}}{h}$ $=\lim\limits_{h\to 0}\dfrac{\frac{-4h}{x(x+h)}}{h}$ $=\lim\limits_{h\to 0}\dfrac{-4h}{x(x+h)}\times\dfrac{1}{h}$ $=-\dfrac{4}{x^2}$	✓ formula ✓ substitution ✓ LCD ✓ simplifying ✓ answer	5
7.2.1	$y=9-12x+4x^2$ $\dfrac{dy}{dx}=-12+8x$	✓ for squaring ✓✓both answers	3

7.2.2	$f(x)=\dfrac{(x-2)(x^2+2x+4)}{(x-2)}$ $=x^2+2x+4$ $f'(x)=2x+2$	✓factorising ✓for simplified f ✓✓answer	4
			[12]

QUESTION 8

8.1	$y = 4x^{\frac{1}{2}}$ $\frac{dy}{dx} = 2x^{-\frac{1}{2}} = \frac{2}{\sqrt{x}}$ $\frac{2}{\sqrt{x}} = 1$ $\sqrt{x} = 2$ $\therefore x = 4$ $and \quad y = 8$ Point $(4;8)$	✓exponential form ✓ derivative ✓derivative equal to 1 ✓ x - value ✓answer	5
8.2		✓for positive y – intercept ✓for turning point at $x = -3$ ✓for turning point at $x = 1$ ✓for shape	4
8.3.1	$SA = 6x^2 + 6x^2 + 4xy + 5xy + 3xy = 3600$ $12xy + 12x^2 = 3600$ $xy + x^2 = 300$ $y = \frac{300 - x^2}{x}$	✓✓ surface area of each side ✓ sum of areas = 3600 ✓ for dividing by 12	4
8.3.2	$V = Area \quad of \quad Base \times height$ $= \left(\frac{1}{2} \times 4x \times 3x\right) \times y$ $= 6x^2\left(\frac{300 - x^2}{x}\right)$ $= 1800x - 6x^3$ $\frac{dV}{dx} = 1800 - 18x^2 = 0$ $100 - x^2 = 0$ $\therefore x = 10$	✓substituting into volume formula ✓simplified expression for volume in terms of x ✓derivative equal to 0 ✓ answer	4
			[17]

QUESTION 9

9.1	$h(3) = 12 - \frac{3}{4} - \frac{3^3}{6} = 6{,}75$ metres	✓answer	1
9.2	$h'(x) = -\frac{1}{4} - \frac{1}{2}t^2$ metres/day $h'(2) = -\frac{1}{4} - \frac{1}{2}(2)^2$ $= -2{,}25$ metres/day Rate of decrease is 2,25l per day	✓ for derivative ✓ for units ✓ substitution ✓ answer and units ✓conclusion	5
			[6]

QUESTION 10

10.1	P(red sock) = 0,5 P(black sock) = 0,3 P(white sock) = 0,2 P(Red and black and white sock) $= P(R) \times P(B) \times P(W)$ $= 0{,}5 \times 0{,}3 \times 0{,}2$ $= 0{,}03$	✓probability of red sock ✓probability of black sock ✓probability of white sock ✓answer	4
10.2	$P(E) = \frac{3}{9} = \frac{1}{3}$ $P(consonant) = \frac{5}{8}$ $P(E, then\ consonant) = \frac{1}{3} \times \frac{5}{8}$ $= \frac{5}{24}$ or 0,21	✓probability of E ✓probability of consonant ✓answer	3
10.3	P(D or K) = P(D) + P(K) 0,64 = P(D) + 3P(D) 0,64 = 4P(D) ∴ P(D) = 0,16	✓ formula ✓substitution ✓ P(D) value	3
10.4.1	No of arrangements = 6! = 720	✓6! ✓answer	2
10.4.2	O and I can be arranged in 2! different ways Remaining letters can be arranged in 4! different ways Total no. of different arrangements having O and I at the start and end = 2! . 4! P(having O and I at the start and end of the word) $= \frac{2!.4!}{720} = \frac{1}{15}$	✓ for 2! ✓ for 4! ✓answer	3
			[15]

TOTAL MARKS: 150